

“Self-tuning” and Conformality

Zurab Kakushadze*

*C.N. Yang Institute for Theoretical Physics
State University of New York, Stony Brook, NY 11794
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Abstract

We consider an infinite-volume brane world setup where a codimension one brane is coupled to bulk gravity plus a scalar field with vanishing potential. The latter is protected by bulk supersymmetry, which is intact even if brane supersymmetry is completely broken as the volume of the extra dimension is infinite. Within this setup we discuss a flat solution with a “self-tuning” property, that is, such a solution exists for a continuous range of values for the brane tension. This infinite-volume solution is free of any singularities, and has the property that the brane cosmological constant is protected by bulk supersymmetry. We, however, also point out that consistency of the coupling between bulk gravity and brane matter generically appears to require that the brane world-volume theory be conformal.

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*E-mail: zurab@insti.physics.sunysb.edu

I. INTRODUCTION

In the Brane World scenario the Standard Model gauge and matter fields are assumed to be localized on branes (or an intersection thereof), while gravity lives in a larger dimensional bulk of space-time [1–12]. The volume of dimensions transverse to the branes is automatically finite if these dimensions are compact. On the other hand, the volume of the transverse dimensions can be finite even if the latter are non-compact. In particular, this can be achieved by using [13] warped compactifications [14] which localize gravity on the brane. A concrete realization of this idea was given in [15].

Recently it was pointed out in [16,17] that, in theories where extra dimensions transverse to a brane have infinite volume [18,19,16,17,20–26], the cosmological constant on the brane might be under control even if brane supersymmetry is completely broken. The key point here is that even if supersymmetry breaking on the brane does take place, it will not be transmitted to the bulk as the volume of the extra dimensions is infinite [16,17]. Thus, at least in principle, one should be able to control some of the properties of the bulk with the unbroken bulk supersymmetry. One then can wonder whether bulk supersymmetry could also control the brane cosmological constant [16,17].

The question whether bulk supersymmetry can control brane cosmological constant was addressed in the codimension one brane world scenarios with an infinite volume extra dimension in [23–25]. Thus, in [23] it was pointed out that if the bulk curvature is constant, bulk supersymmetry does not control the brane cosmological constant. However, as was pointed out in [24,25], bulk supersymmetry might control the brane cosmological constant if the bulk curvature is not constant. An example of such a setup was given in [24]. Thus, in the setup of [24] non-constant bulk curvature is parametrized by a bulk scalar ϕ which has a non-trivial profile in the extra dimension due to the fact that the corresponding scalar potential depends non-trivially on ϕ . As was argued in [24], bulk supersymmetry in this setup indeed controls the brane cosmological constant. However, as was argued in [25], the brane world-volume theory in this setup must be conformal in order to have consistent coupling between bulk gravity and brane matter.

In this paper we study a somewhat modified setup. In particular, we consider a model where the bulk scalar potential vanishes. In this case the model has a self-tuning property, that is, a flat solution exist for a continuous range of values for the brane tension. However, unlike the setup of [27], in the model we discuss in this paper the volume of the extra dimension is infinite, and, in particular, the background is free of any naked singularities. Moreover, since the volume of the extra dimension is infinite, bulk supersymmetry protects the self-tuning property. In particular, vanishing of the bulk scalar potential can be achieved without any fine tuning by imposing unbroken R -symmetry in the bulk.

However, the setup we discuss in this paper also has the property that consistency of the coupling between brane matter and bulk gravity requires that the brane world-volume theory be conformal. This property is due to the fact that the brane tension in this model is non-vanishing, and, as was recently pointed out in [28], one cannot gauge away the graviscalar in such backgrounds as the corresponding diffeomorphisms are explicitly broken by the presence of such a brane. (The brane tension in this model is negative, so the brane must be placed at a \mathbf{Z}_2 orbifold fixed point to avoid ghosts in the brane world-volume theory.) If the brane world-volume theory is not conformal, one then expects various terms involving

the graviscalar to be generated in the brane world-volume action at the quantum level. Such terms, however, lead to inconsistencies in the coupling between brane matter and bulk gravity.

The remainder of this paper is organized as follows. In section II we discuss the setup of this paper. In section III we study small fluctuations around the solution in the presence of brane matter sources, and discuss the requirement that the brane matter be conformal. Section IV contains concluding remarks.

II. THE MODEL

Consider the model with the following action (more precisely, here we give the part of the action relevant for the subsequent discussions):

$$S = - \int_{\Sigma} d^{D-1}x \sqrt{-\hat{G}} f + M_P^{D-2} \int d^Dx \sqrt{-G} \left[R - \frac{4}{D-2} (\nabla\phi)^2 \right]. \quad (1)$$

For calculational convenience we will keep the number of space-time dimensions D unspecified. In (1) M_P is the D -dimensional (reduced) Planck scale. The $(D-1)$ -dimensional hypersurface Σ , which we will refer to as the brane, is the $z=0$ slice of the D -dimensional space-time, where $z \equiv x^D$. Next,

$$\hat{G}_{\mu\nu} \equiv \delta_{\mu}^M \delta_{\nu}^N G_{MN} \Big|_{z=0}, \quad (2)$$

where the capital Latin indices $M, N, \dots = 1, \dots, D$, while the Greek indices $\mu, \nu, \dots = 1, \dots, (D-1)$. The quantity f is the brane tension. More precisely, there might be various (massless and/or massive) fields (such as scalars, fermions, gauge vector bosons, *etc.*), which we will collectively denote via Φ^i , localized on the brane. Then $f = f(\Phi^i, \nabla_{\mu}\Phi^i, \dots)$ generally depends on the vacuum expectation values of these fields as well as their derivatives. In the following we will assume that the expectation values of the Φ^i fields are dynamically determined, independent of the coordinates x^{μ} , and consistent with $(D-1)$ -dimensional general covariance. The quantity f is then a constant which we identify as the brane tension. The bulk fields are given by the metric G_{MN} , a single real scalar field ϕ , as well as other fields (whose expectation values we assume to be vanishing) which would appear in a concrete supergravity model (for the standard values of D). The scalar potential for the field ϕ is assumed to be vanishing. Note that this can be achieved without fine-tuning as the bulk is supersymmetric.

To proceed further, we will need equations of motion following from the action (1). These are given by:

$$\frac{8}{D-2} \nabla^2 \phi = \frac{\sqrt{-\hat{G}}}{\sqrt{-G}} \tilde{f}_{\phi} \delta(z), \quad (3)$$

$$R_{MN} - \frac{1}{2} G_{MN} R = \frac{4}{D-2} \left[\nabla_M \phi \nabla_N \phi - \frac{1}{2} G_{MN} (\nabla\phi)^2 \right] - \frac{1}{2} \frac{\sqrt{-\hat{G}}}{\sqrt{-G}} \delta_M^{\mu} \delta_N^{\nu} \hat{G}_{\mu\nu} \tilde{f} \delta(z). \quad (4)$$

Here $\tilde{f} \equiv f/M_P^{D-2}$, and the subscript ϕ indicates derivative w.r.t. ϕ (note that generally f depends on ϕ , more precisely, on its value at $z=0$).

Here we are interested in studying possible solutions to these equations which are consistent with $(D - 1)$ -dimensional general covariance. That is, we will be looking for solutions with the warped metric of the following form:

$$ds_D^2 = \exp(2A) \left[\tilde{g}_{\mu\nu} dx^\mu dx^\nu + dz^2 \right] , \quad (5)$$

where the warp factor A and the scalar field ϕ , which are functions of z , are independent of the coordinates x^μ , and the $(D - 1)$ -dimensional metric $\tilde{g}_{\mu\nu}$ is independent of z . With this ansatz, we have the following equations of motion for ϕ and A :

$$\frac{8}{D-2} [\phi'' + (D-2)A'\phi'] = \exp(A) \tilde{f}_\phi \delta(z) , \quad (6)$$

$$(D-1)(D-2)(A')^2 - \frac{4}{D-2}(\phi')^2 - \frac{D-1}{D-3}\tilde{\Lambda} = 0 , \quad (7)$$

$$(D-2) [A'' - (A')^2] + \frac{4}{D-2}(\phi')^2 + \frac{1}{D-3}\tilde{\Lambda} = -\frac{1}{2}\exp(A) \tilde{f}_\phi \delta(z) , \quad (8)$$

where a prime indicates derivative w.r.t. z . Next, $\tilde{\Lambda}$ is independent of x^μ and z . In fact, it is nothing but the cosmological constant of the $(D - 1)$ -dimensional manifold, which is therefore an Einstein manifold, corresponding to the hypersurface Σ . Our normalization of $\tilde{\Lambda}$ is such that the $(D - 1)$ -dimensional metric $\tilde{g}_{\mu\nu}$ satisfies Einstein's equations:

$$\tilde{R}_{\mu\nu} - \frac{1}{2}\tilde{g}_{\mu\nu}\tilde{R} = -\frac{1}{2}\tilde{g}_{\mu\nu}\tilde{\Lambda} . \quad (9)$$

Here we note that in the bulk (that is, for $z \neq 0$) one of the second order equations is automatically satisfied once the first order equation (7) as well as the other second order equation are satisfied. As usual, this is a consequence of Bianchi identities.

A. Bulk Supersymmetry and Brane Cosmological Constant

In the following we will be interested in solutions to the above equations of motion such that the volume of the z dimension is infinite. Consistent solutions of this type exist for vanishing as well as non-vanishing brane cosmological constant $\tilde{\Lambda}$. We will, however, assume that the bulk is supersymmetric. In fact, as was pointed out in [16,17], if the volume of the extra dimension is infinite, bulk supersymmetry is intact even if brane supersymmetry is completely broken. Then, as was pointed out in [24], if the bulk curvature is not constant¹, then bulk supersymmetry might control the brane cosmological constant. In the above model bulk curvature is not constant as long as the ϕ field has a non-trivial profile. Then it is not difficult to check that bulk supersymmetry indeed controls the brane cosmological constant. In fact, this follows from the bulk Killing spinor equations (following from the requirement that variations of the superpartner λ of ϕ and the gravitino ψ_M vanish under the corresponding supersymmetry transformations), which in such backgrounds reduce to:

¹As was pointed out in [23], if the bulk curvature is constant, then bulk supersymmetry does not control the brane cosmological constant.

$$\phi' = \alpha W_\phi \exp(A) , \quad (10)$$

$$A' = \beta W \exp(A) , \quad (11)$$

where W is the superpotential,

$$\alpha \equiv \eta \frac{\sqrt{D-2}}{2} , \quad \beta \equiv -\eta \frac{2}{(D-2)^{3/2}} , \quad (12)$$

and $\eta = \pm 1$.

Note that the system of equations (10) and (11) is compatible with the system of equations (6), (7) and (8) if and only if $\tilde{\Lambda} = 0$, and

$$W = C \exp(\epsilon \gamma \phi) , \quad (13)$$

where C is a constant, $\epsilon = \pm 1$, and

$$\gamma \equiv \frac{2\sqrt{D-1}}{D-2} . \quad (14)$$

In fact, (13) is simply the statement that the scalar potential, which is given by the familiar expression $V = W_\phi^2 - \gamma^2 W^2$, vanishes in the model defined in (1).

Thus, bulk supersymmetry (note that a solution to (10) and (11) preserves 1/2 of the supersymmetries compared with a flat solution) is preserved if and only if the brane cosmological constant vanishes. We therefore conclude that even if brane supersymmetry is broken, bulk supersymmetry, which remains unbroken as the volume of the transverse dimension is infinite, ensures that the brane cosmological constant still vanishes in the model defined in (1).

B. A Self-tuning Solution with Infinite Volume

In this subsection we would like to discuss a solution of (6), (7) and (8) with the self-tuning property. That is, this solution has vanishing brane cosmological constant, which, moreover, exists for a continuous range of values for the brane tension.

Thus, consider the following solution:

$$\phi(z) = \kappa \frac{\sqrt{D-1}}{2} \ln \left[\frac{|z|}{\Delta} + 1 \right] + \phi_0 , \quad (15)$$

$$A(z) = \frac{1}{D-2} \ln \left[\frac{|z|}{\Delta} + 1 \right] + A_0 , \quad (16)$$

where $\kappa = \pm 1$, and Δ is a *positive* quantity, which is related to the brane tension via

$$\Delta = -4/\tilde{f} . \quad (17)$$

That is, in this solution the brane tension f is assumed to be negative. Moreover, we have an additional condition on f given by (ϵ and γ were defined in the previous subsection)

$$f_\phi(\phi_0) = -\kappa \gamma f(\phi_0) . \quad (18)$$

Note that this solution is non-singular, and the volume of the extra dimension is infinite as the integral

$$\int dz \exp(DA) \tag{19}$$

diverges. Moreover, this solution preserves 1/2 of the original supersymmetries in the bulk. The brane cosmological constant in this solution is vanishing. Most importantly, such a solution exists for a continuous range of values for the brane tension (namely, for any negative brane tension) - indeed, changing the brane tension amounts to appropriately changing the parameter Δ . Note that the immaterial integration constant $A_0 \equiv A(0)$ is not determined and can be set to zero, and we will do so in the following. As to the integration constant $\phi_0 \equiv \phi(0)$, it is fixed by (18). Thus, the condition (18) is not a fine-tuning, but rather fixes the value of the scalar field ϕ on the brane².

The above solution has a \mathbf{Z}_2 symmetry w.r.t. the reflection $z \rightarrow -z$. The reason why we are focusing on this solution is that we must actually consider the orbifolded version of this solution so that the geometry of the z dimension is \mathbf{R}/\mathbf{Z}_2 (and not \mathbf{R}). The reason for this is that the brane tension is negative, and unless the brane is an “end-of-the-world” brane stuck at the \mathbf{Z}_2 fixed point located at $z = 0$, the brane world volume theory would suffer from ghosts. In the following we will therefore adapt the orbifolded version of this solution.

Here the following remark is in order. The self-tuning property of the above solution does not imply that there are no solutions with non-zero cosmological constant. In fact, it is not difficult to check that, even if we confine to \mathbf{Z}_2 symmetric solutions (so that we can avoid ghosts by considering orbifolded versions thereof), solutions with $\tilde{\Lambda} \neq 0$ do exist. In particular, for such solutions the analog of the condition (18) contains the cosmological constant $\tilde{\Lambda}$. However, in the above setup what ensures vanishing of the brane cosmological constant is bulk supersymmetry, which is intact even if brane supersymmetry is completely broken as the volume of the extra dimension is infinite. The condition (18) then fixes ϕ_0 .

Note that in the above discussion it is important that the volume of the transverse dimension is infinite. Had it been finite, there would be nothing there to protect the brane cosmological constant (except if supersymmetry is unbroken both on the brane and in the bulk). Thus, consider solutions of the above type in the cases where the brane tension is positive. Then Δ defined above is negative, and we have a naked singularity at a finite distance from the brane. Such solutions were originally discussed in [27], where the space in the z direction was cut off at the singularities, so that the solution appears to have a finite volume. The self-tuning property, however, would be lost in finite volume cases. At any rate, as was pointed out in [29], cutting off the space at singularities arising in such solutions is not consistent³.

²Here one assumes that $f(\phi)$ is such that there exists ϕ_0 such that (18) is satisfied. Otherwise, \mathbf{Z}_2 symmetric solutions with $\tilde{\Lambda} = 0$ do not exist.

³Some possibilities for resolving such singularities were discussed in [30,31].

III. BULK GRAVITY AND BRANE MATTER

In this section we would like to study gravitational interactions between sources localized on the brane. To do this, let us start from the action (1), and study small fluctuations of the metric G_{MN} and the scalar field ϕ , which we will denote via h_{MN} and φ , respectively, around the self-tuning solution with infinite volume discussed in the previous section.

In the following, instead of metric fluctuations h_{MN} , it will be convenient to work with \tilde{h}_{MN} defined via

$$h_{MN} = \exp(2A)\tilde{h}_{MN} . \quad (20)$$

It is not difficult to see that in terms of \tilde{h}_{MN} the D -dimensional diffeomorphisms

$$\delta h_{MN} = \nabla_M \xi_N + \nabla_N \xi_M \quad (21)$$

are given by the following gauge transformations (the capital Latin indices are lowered and raised with the flat D -dimensional Minkowski metric η_{MN} and its inverse):

$$\delta \tilde{h}_{MN} = \partial_M \tilde{\xi}_N + \partial_N \tilde{\xi}_M + 2A' \eta_{MN} \tilde{\xi}_S n^S . \quad (22)$$

Here for notational convenience we have introduced a unit vector n^M with the following components: $n^\mu = 0$, $n^D = 1$.

Note that, as was pointed out in [28], the presence of the non-zero tension brane explicitly breaks the full D -dimensional diffeomorphism invariance (22) to a smaller subset of gauge transformations given by (22) with the restrictions that

$$\tilde{\xi}_D(z=0) = 0 , \quad \tilde{\xi}'_\mu(z=0) = 0 . \quad (23)$$

There is, however, a further restriction on the unbroken gauge transformations coming from the fact that the brane is stuck at the orbifold fixed point. This restriction reads:

$$\tilde{\xi}'_D(z=0) = 0 . \quad (24)$$

It is then not difficult to see that if $\rho(z=0)$ is non-zero, then we cannot gauge it away even just on the brane, where $\rho \equiv \tilde{h}_{DD}$. However, we can gauge $A_\mu \equiv \tilde{h}_{\mu D}$ away everywhere as long as $A_\mu(z=0) = 0$, which is indeed the case as we will see in the next subsection.

A. Equations of Motion

To proceed further, we need equations of motion for \tilde{h}_{MN} and φ . Let us assume that we have matter localized on the brane, and let the corresponding conserved energy-momentum tensor be $T_{\mu\nu}$:

$$\partial^\mu T_{\mu\nu} = 0 . \quad (25)$$

The graviton field $\tilde{h}_{\mu\nu}$ couples to $T_{\mu\nu}$ via the following term in the action (recall that we have set $A(0) = 0$):

$$S_{\text{int}} = \int_{\Sigma} d^{D-1}x \left[\frac{1}{2} T_{\mu\nu} \tilde{h}^{\mu\nu} + \frac{8}{D-2} \Theta \varphi \right] , \quad (26)$$

where we have also included the corresponding coupling of φ to the brane matter. Next, starting from the action $S + S_{\text{int}}$ we obtain the following linearized equations of motion for \tilde{h}_{MN} and φ :

$$\begin{aligned} & \left\{ \partial_S \partial^S \tilde{h}_{MN} + \partial_M \partial_N \tilde{h} - \partial_M \partial^S \tilde{h}_{SN} - \partial_N \partial^S \tilde{h}_{SM} - \eta_{MN} [\partial_S \partial^S \tilde{h} - \partial^S \partial^R \tilde{h}_{SR}] \right\} + \\ & (D-2)A' \left\{ [\partial_S \tilde{h}_{MN} - \partial_M \tilde{h}_{NS} - \partial_N \tilde{h}_{MS}] n^S + \eta_{MN} [2\partial^R \tilde{h}_{RS} - \partial_S \tilde{h}] n^S \right\} = \\ & \frac{8}{D-2} \phi' [\eta_{MN} \partial_S \varphi n^S - \partial_M \varphi n_N - \partial_N \varphi n_M] - M_P^{2-D} \tilde{T}_{MN} \delta(z) , \end{aligned} \quad (27)$$

$$\partial_S \partial^S \varphi + (D-2)A' \partial_S \varphi n^S - \frac{1}{2} \phi' [2\partial^R \tilde{h}_{RS} - \partial_S \tilde{h}] n^S = -M_P^{2-D} \tilde{\Theta} \delta(z) , \quad (28)$$

where $\tilde{h} \equiv \tilde{h}_M^M$, $\tilde{T}_{MN} \equiv T_{MN} + T_{MN}^{\text{brane}}$, $\tilde{\Theta} \equiv \Theta + \Theta^{\text{brane}}$. Here T_{MN}^{brane} and Θ^{brane} are the corresponding brane contributions (which are linear in \tilde{h}_{MN} and φ) coming from the first term in (1)⁴. Thus, we have:

$$T_{\mu\nu}^{\text{brane}} = -\eta_{\mu\nu} \left[f_{\phi} \varphi + \frac{1}{2} f \rho \right] , \quad (29)$$

$$T_{\mu D}^{\text{brane}} = A_{\mu} f , \quad (30)$$

$$T_{DD}^{\text{brane}} = 0 , \quad (31)$$

$$\Theta^{\text{brane}} = -\frac{D-2}{8} \left[f_{\phi\phi} \varphi + \frac{1}{2} f_{\phi} \rho \right] . \quad (32)$$

Note that *a priori* $\partial^{\mu} \tilde{T}_{\mu\nu} \neq 0$.

It is not difficult to see that the r.h.s. of the (μD) component of (27) does not contain any terms with the second derivative w.r.t. z . This then implies that, to have a consistent solution, we must have $A_{\mu}(z=0) = 0$. As we have already mentioned, we can then gauge A_{μ} away everywhere, and we will adapt this gauge in the following. We then have the following equations of motion (the Greek indices are lowered and raised with the flat $(D-1)$ -dimensional Minkowski metric and its inverse):

$$\begin{aligned} & \left\{ \partial_{\sigma} \partial^{\sigma} H_{\mu\nu} + \partial_{\mu} \partial_{\nu} H - \partial_{\mu} \partial^{\sigma} H_{\sigma\nu} - \partial_{\nu} \partial^{\sigma} H_{\sigma\mu} - \eta_{\mu\nu} [\partial_{\sigma} \partial^{\sigma} H - \partial^{\sigma} \partial^{\rho} H_{\sigma\rho}] \right\} + \\ & \left\{ H''_{\mu\nu} - \eta_{\mu\nu} H'' + (D-2)A' [H'_{\mu\nu} - \eta_{\mu\nu} H'] \right\} + \\ & \left\{ \partial_{\mu} \partial_{\nu} \rho - \eta_{\mu\nu} \partial_{\sigma} \partial^{\sigma} \rho + \eta_{\mu\nu} (D-2)A' \rho' \right\} - \frac{8}{D-2} \eta_{\mu\nu} \phi' \varphi' = -M_P^{2-D} \tilde{T}_{\mu\nu} \delta(z) , \end{aligned} \quad (33)$$

$$[\partial^{\mu} H_{\mu\nu} - \partial_{\nu} H]' + (D-2)A' \partial_{\nu} \rho - \frac{8}{D-2} \phi' \partial_{\nu} \varphi = 0 , \quad (34)$$

⁴If the brane world-volume theory is not conformal, then we can *a priori* expect additional contributions arising due to quantum corrections. We will discuss effects of such contributions in the next subsection, which will lead us to the conclusion that to maintain consistent coupling between bulk gravity and brane matter the brane world-volume theory should be conformal.

$$-[\partial^\mu \partial^\nu H_{\mu\nu} - \partial^\mu \partial_\mu H] + (D-2)A'H' - \frac{8}{D-2}\phi'\phi' = 0 , \quad (35)$$

$$\partial^\mu \partial_\mu \varphi + \varphi'' + (D-2)A'\varphi' + \frac{1}{2}\phi'[H' - \rho'] = -M_P^{2-D}\tilde{\Theta}\delta(z) , \quad (36)$$

where $H \equiv H_\mu^\mu$. Here we note that, as usual, not all of the above equations are independent.

The above system of equations can be analyzed much along the lines of [25,28], so we will skip the details and simply give the answer. Thus, after some straightforward (albeit tedious) computations one can show that for a consistent solution to the above equations to exist we must have⁵ ($\kappa = \pm 1$ was defined in section II)

$$\tilde{\Theta} = -\frac{\kappa}{4\sqrt{D-1}}\tilde{T} , \quad (37)$$

where $T \equiv T_\mu^\mu$. The corresponding solution is then given by:

$$\rho = H , \quad \varphi = \kappa \frac{D-2}{4\sqrt{D-1}}H , \quad (38)$$

$$H_{\mu\nu}(p, z) = M_P^{2-D} \left[T_{\mu\nu}(p) - \frac{1}{D-2}\eta_{\mu\nu}T(p) \right] \Omega(p, z) , \quad (39)$$

where Ω is the solution to the following equation ($p^2 \equiv p^\mu p_\mu$)

$$\Omega''(p, z) + (D-2)A'\Omega'(p, z) - p^2\Omega(p, z) = -\delta(z) \quad (40)$$

subject to the boundary conditions (for $p^2 > 0$)

$$\Omega(p, z \rightarrow \pm\infty) = 0 . \quad (41)$$

In the above expressions we have performed a Fourier transform for the coordinates x^μ (the corresponding momenta are p^μ), and Wick rotated to the Euclidean space (where the propagator is unique).

Note that for the above solution we have $T_{\mu\nu}^{\text{brane}} = 0$, so that $\tilde{T}_{\mu\nu} = T_{\mu\nu}$. The tensor structure of the graviton propagator following from (39) is that of the D -dimensional massless graviton (and not of the $(D-1)$ -dimensional one), which is in accord with the fact that the volume of the extra dimension is infinite. As to the scalar degrees of freedom ρ and φ , they couple to the trace of the energy-momentum tensor on the brane, that is, the corresponding couplings are non-vanishing as long as the brane world-volume theory is not conformal.

B. Conformality

The fact that ρ cannot be gauged away has important implications. Thus, suppose that the brane world-volume theory is not conformal. Then quantum corrections on the brane will generically generate various terms in the brane world-volume [22,26]. These terms are *a*

⁵Here we note that this condition is analogous to that arising in the setup studied in [25].

priori arbitrary except that they must respect the gauge symmetries of the system. On the brane these symmetries are given by the $(D - 1)$ -dimensional diffeomorphism under which ρ and φ do not transform. Thus, for instance, consider such terms that do not contain any derivatives. The most general corrections of this type can be written as follows:

$$S_1 = -M_*^{D-1} \int_{\Sigma} dx^{D-1} F(\rho, \varphi) , \quad (42)$$

where $F(\rho, \varphi)$ is some dimensionless function of ρ and φ , and M_* denotes a mass scale in the brane world-volume theory which determines the size of quantum corrections (this can be the cut-off scale of the brane world-volume theory, or the mass scale of some heavy fields localized on the brane). Note that the ρ -independent part of $F(\rho, \varphi)$, that is, $F(0, \varphi)$ corresponds to renormalizing $f(\phi)$ in the first term in (1). Due to the self-tuning property of the solution we are discussing here, such a renormalization does not affect the qualitative features of the solution as long as the equation

$$\hat{f}_{\phi}(\phi_0) = -\kappa\gamma\hat{f}(\phi_0) , \quad (43)$$

where \hat{f} is the renormalized counterpart of f , has a solution such that $\hat{f}(\phi_0)$ is negative. However, the ρ -dependent part of $F(\rho, \varphi)$, which is generically expected to be non-vanishing as long as the brane world-volume theory is not conformal, introduces an inconsistency into the system. Thus, note that the r.h.s. of (35) now contains a non-vanishing source term proportional to $\delta(z)$. Since the l.h.s. of this equation does not contain any second derivatives w.r.t. z , we conclude that we no longer have a consistent solution.

Thus, as we see, the fact that ρ cannot be gauged away on the brane necessitates fine-tuning at the quantum level to preserve consistent coupling between bulk gravity and brane matter as long as the brane world-volume theory is not conformal. On the other hand, if we assume that the brane world-volume theory is conformal, the undesirable terms discussed above will not be generated. Indeed, in this case we have $T = 0$, and

$$\rho = \varphi = 0 , \quad (44)$$

$$H_{\mu\nu}(p, z) = M_P^{2-D} T_{\mu\nu}(p) \Omega(p, z) . \quad (45)$$

This then implies that the coupling of the scalar field φ to the brane matter also vanishes: $\Theta = 0$. Quantum corrections then will not generate any dangerous terms (including those containing derivatives), and the background as well as the coupling between brane matter and bulk gravity should remain consistent at the quantum level.

IV. REMARKS

The requirement that the brane world-volume theory be conformal has already appeared in a somewhat different context in [24,25]. There too conformality is required to maintain consistent coupling between brane matter and bulk gravity. In [24,25] as well as in the model discussed in this paper it is important that the extra dimension has infinite volume (and also that the bulk is supersymmetric). Thus, as was recently pointed out in [28], similar

inconsistencies are expected⁶ to arise at the quantum level in, say, the original Randall-Sundrum model [15]. There, however, the brane world-volume theory cannot be conformal as long as gravity is localized⁷.

It would be interesting to understand whether consistency of the coupling between brane matter and bulk gravity imposes any constraints on the brane world-volume theory in the cases of higher codimension brane world scenarios with infinite volume extra dimensions [26]. In particular, if in some cases the brane world-volume theory must be conformal, it would be interesting to understand if there is a relation to [34].

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⁶Some of the terms in the brane world-volume action that would lead to such inconsistencies in this context were discussed in [28], and such terms indeed seem to be present [32].

⁷As was pointed out in [29,33], gravity in such backgrounds is generically expected to be delocalized due to higher curvature bulk contributions.

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